# 1: “`Category Theory.zip`” and Simplicial Sets

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These notes roughly follow the contents of [Wag25, §1] and [Lan21, §1]

## 1 Adjunctions and equivalences

**Definition 1.1.** Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be functors. Write $L \dashv R$ if there exists an iso

$$\mathrm{Hom}_{\mathcal{D}}(L-,-) \cong \mathrm{Hom}_{\mathcal{C}}(-,R-)$$

in $\mathrm{Fun}(\mathcal{C}^{\mathrm{op}} \times \mathcal{D}, \mathbf{Set})$.

**Lemma 1.2.** *$L\colon \mathcal{C} \longrightarrow \mathcal{D}$ has a right adjoint $\iff$ every presheaf*

$$\mathrm{Hom}_{\mathcal{D}}(L-,d)\colon \mathcal{C}^{\mathrm{op}} \longrightarrow \mathbf{Set}$$

*is representable. (A dual result holds for left adjoints.)*

*Proof.* $\Rightarrow$ is clear, so we show $\Leftarrow$. Identify $\mathrm{Hom}_{\mathcal{D}}(L-,-)\colon \mathcal{C}^{\mathrm{op}} \times \mathcal{D} \longrightarrow \mathbf{Set}$ with $F\colon \mathcal{D} \longrightarrow \mathrm{PSh}(\mathcal{C})$. By assumption, all $Fd \in \mathrm{EssIm}(\text{よ})$. Let $\text{よ}^{-1}\colon \mathrm{EssIm}(\text{よ}) \longrightarrow \mathcal{C}$ be a quasi-inverse. The composite

$$\begin{tikzcd} \mathcal{D} \arrow[r, "F"] \arrow[rr, bend right, dashed, "R"'] & \mathrm{EssIm}(\text{よ}) \arrow[r, "\text{よ}^{-1}"] & \mathcal{C} \end{tikzcd}$$

defines a right adjoint $R$. $\square$

**Remark 1.3.** Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be adjoints ($L \dashv R$). The iso

$$\mathrm{Hom}_{\mathcal{D}}(Lc, Lc) \cong \mathrm{Hom}_{\mathcal{C}}(c, RLc)$$

sends $\mathrm{id}_{Lc}$ to $\eta_c\colon c \longrightarrow RLc$. This defines the unit $\eta\colon \mathrm{id} \Longrightarrow RL$. The iso

$$\mathrm{Hom}_{\mathcal{C}}(Rd, Rd) \cong \mathrm{Hom}_{\mathcal{D}}(LRd, d)$$

sends $\mathrm{id}_{Rc}$ to $\varepsilon_d\colon LRd \longrightarrow d$. This defines the counit $\varepsilon\colon LR \Longrightarrow \mathrm{id}$. They satisfy the triangle identities.

$$\begin{tikzcd} L \arrow[r, "L\eta"] \arrow[d, "\mathrm{id}"'] & LRL \arrow[dl, "\varepsilon L"] \\ L & \end{tikzcd} \qquad \begin{tikzcd} R & RLR \arrow[l, "R\varepsilon"'] \\ R \arrow[u, "\mathrm{id}"] \arrow[ur, "\eta R"'] & \end{tikzcd}$$

**Exercise 1.4.** Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be functors. TFAE:

1. $L \dashv R$
2. There exist $\eta\colon \mathrm{id} \Longrightarrow RL$ and $\varepsilon\colon LR \Longrightarrow \mathrm{id}$ that satisfy the triangle identities.

**Corollary 1.5.** *Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be adjoints, $\mathcal{J}$ any category. Then we have adjunctions*

$$L_*\colon \mathrm{Fun}(\mathcal{J}, \mathcal{C}) \leftrightarrows \mathrm{Fun}(\mathcal{J}, \mathcal{D}) : R_*$$
$$R^*\colon \mathrm{Fun}(\mathcal{C}, \mathcal{J}) \leftrightarrows \mathrm{Fun}(\mathcal{D}, \mathcal{J}) : L^*$$

*Proof.* It suffices to construct the unit and counit. These are inherited from $L \dashv R$. □

**Lemma 1.6.** *Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be an adjunction.*

1. *$L$ is fully faithful $\iff$ $\eta\colon \mathrm{id} \Longrightarrow RL$ is an iso.*
2. *If $L$ is fully faithful and $R$ reflects isos, then $L \dashv R$ is an adjoint equivalence.*

*Proof.* Let $x, y \in \mathcal{C}$. By definition of $\eta$ (see Remark 1.3) we can factor $\text{よ}(\eta_y)_x$ via

$$\begin{tikzcd} \mathrm{Hom}_{\mathcal{C}}(x, y) \arrow[r, "L"] \arrow[rr, dashed, bend right, "\text{よ}(\eta_y)_x"'] & \mathrm{Hom}_{\mathcal{D}}(Lx, Ly) \arrow[r, "\text{adjunction}", "\cong"'] & \mathrm{Hom}_{\mathcal{C}}(x, RLy) \end{tikzcd}$$

It follows that

$$\begin{aligned} L \text{ is fully faithful} &\iff \text{all } \text{よ}(\eta_y)_x \text{are bijections} \\ &\iff \text{all } \text{よ}(\eta_y) \text{ are isos} \\ &\iff \eta \text{ is an iso.} \end{aligned}$$

This shows 1. To see 2. consider the triangle identity

$$\begin{tikzcd} R \arrow[r, "\eta R"] \arrow[rr, bend right, "\mathrm{id}"'] & RLR \arrow[r, "R\varepsilon"] & R \end{tikzcd}$$

and note that if $\eta$ is an iso, then so is $\eta R$. Hence $R\varepsilon$ is an iso, and thus $\varepsilon$ is an iso. □

# 2 Limits and colimits

**Definition 2.1.** Let $\mathcal{J}$ be a category. Denote by $\mathrm{const}\colon \mathcal{C} \longrightarrow \mathrm{Fun}(\mathcal{J}, \mathcal{C})$ the functor that sends $c \in \mathcal{C}$ to the constant diagram.
A limit of $F\colon \mathcal{J} \longrightarrow \mathcal{C}$ is an object $\lim F \in \mathcal{C}$ satisfying

$$\mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathcal{C})}(\mathrm{const}_{(-)}, F) \cong \mathrm{Hom}_{\mathcal{C}}(-, \lim F).$$

A colimit of $F$ is an object $\operatorname{colim} F \in \mathcal{C}$ satisfying

$$\mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathcal{C})}(F, \mathrm{const}_{(-)}) \cong \mathrm{Hom}_{\mathcal{C}}(\operatorname{colim} F, -).$$

**Remark 2.2.** If all diagrams $F\colon \mathcal{J} \longrightarrow \mathcal{C}$ admit (co)limits, applying Lemma 1.2 yields

$$\operatorname*{colim}_{\mathcal{J}} \dashv \mathrm{const} \dashv \lim_{\mathcal{J}}$$

for free. In particular, taking (co)limits is functorial.

**Lemma 2.3.** *Let $L\colon \mathcal{C} \leftrightarrows \mathcal{D} : R$ be adjoints.*

1. *$L$ preserves colimits.*
2. *$R$ preserves limits.*

*Proof.* Suppose $F\colon \mathcal{J} \longrightarrow \mathcal{C}$ has a colimit. Using $L_* \dashv R_*$ and Lemma 1.2 we obtain

$$\begin{aligned}
\mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathcal{D})}(LF, \mathrm{const}_{(-)}) &\cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathcal{C})}(F, R\,\mathrm{const}_{(-)}) \\
&\cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathcal{C})}(F, \mathrm{const}_{R(-)}) \\
&\cong \mathrm{Hom}_{\mathcal{C}}(\operatorname{colim} F, R-) \\
&\cong \mathrm{Hom}_{\mathcal{D}}(L \operatorname{colim} F, -).
\end{aligned}$$

The limit case is analogous. □

**Lemma 2.4.** *Suppose all $\mathcal{J}$-shaped (co)limits exist in $\mathcal{D}$. Then the same holds for* $\mathrm{Fun}(\mathcal{C}, \mathcal{D})$ *and (co)limits are computed pointwise.*

*Proof.* We have a commutative diagram

$$\begin{tikzcd}
\mathrm{Fun}(\mathcal{C},\mathcal{D}) \arrow[r, "\mathrm{const}^F"] \arrow[rr, bend right, "\mathrm{const}^{\mathcal{C}}_*"'] & \mathrm{Fun}(\mathcal{J},\mathrm{Fun}(\mathcal{C},\mathcal{D})) \arrow[r, "\cong"] & \mathrm{Fun}(\mathcal{C},\mathrm{Fun}(\mathcal{J},\mathcal{D}))
\end{tikzcd}$$

By Corollary 1.5 we have $\mathrm{const}^{\mathcal{C}}_* \dashv \lim_*$. Hence, $\mathrm{const}^F$ has a right adjoint. The pointwise condition comes from the top-right iso. The case for colimits is analogous. □

**Corollary 2.5.** *Hom-functors preserve limits.*

*Proof.* Suppose $F\colon \mathcal{J} \longrightarrow C$ has a limit. For $c \in C$ we have natural isos

$$\begin{aligned}
\mathrm{Hom}_C(c, \lim F) &\cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},C)}(\mathrm{const}_c, F) \\
&\cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathrm{PSh}(C))}(よ\,\mathrm{const}_c, よF) \\
&\cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{J},\mathrm{PSh}(C))}(\mathrm{const}_{よ(c)}, よF) \\
&\cong \mathrm{Hom}_{\mathrm{PSh}(C)}(よ(c), \lim(よF)) \\
&\cong \lim \mathrm{Hom}_C(c, F-).
\end{aligned}$$

where we use that **Set** is complete and limits in $\mathrm{PSh}(C)$ are pointwise. The $\mathrm{Hom}_C(-,c)$ case is an exercise. ☐

# 3 Kan extensions

**Remark 3.1.** We will now develop the machinery that provides the adjunctions

$$|\cdot|\colon \mathbf{sSet} \leftrightarrows \mathbf{Top} : \mathrm{Sing},\ \ h\colon \mathbf{sSet} \leftrightarrows \mathbf{Cat} : N,\ \ (-)\times X\colon \mathbf{sSet} \leftrightarrows \mathbf{sSet}\colon\ \mathrm{F}(X,-)$$

for free. To this end we have to study Kan extensions along よ.

Suppose we are given functors $F$ and $K$ as follows.

$$\begin{tikzcd}
\mathcal{C} \arrow[r, "F"] \arrow[d, "K"'] & \mathcal{D} \\
\mathcal{E} &
\end{tikzcd}$$

Our goal is to “fill this horn” in **Cat** universally by a 2-cell (not necessarily as a commutative triangle) that provides a weak extension of $F$. Since natural transformations are oriented, there are two approaches.

$$\begin{tikzcd}
\mathcal{C} \arrow[r, "F"] \arrow[d, "K"'] \arrow[dr, phantom, "\Downarrow"] & \mathcal{D} \\
\mathcal{E} \arrow[ur, dashed] &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathcal{C} \arrow[r, "F"] \arrow[d, "K"'] \arrow[dr, phantom, "\Uparrow"] & \mathcal{D} \\
\mathcal{E} \arrow[ur, dashed] &
\end{tikzcd}$$

**Definition 3.2.** The left Kan extension of $F\colon \mathcal{C} \longrightarrow \mathcal{D}$ along $K\colon \mathcal{C} \longrightarrow \mathcal{E}$ is defined via

$$\mathrm{Hom}_{\mathrm{Fun}(\mathcal{C},\mathcal{D})}(F, K^*(-)) \cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{E},\mathcal{D})}(\mathrm{Lan}_K F, -).$$

The right Kan extension of $F\colon \mathcal{C} \longrightarrow \mathcal{D}$ along $K\colon \mathcal{C} \longrightarrow \mathcal{E}$ is defined via

$$\mathrm{Hom}_{\mathrm{Fun}(\mathcal{C},\mathcal{D})}(K^*(-), F) \cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{E},\mathcal{D})}(-, \mathrm{Ran}_K F).$$

**Remark 3.3.** If every $F$ admits Kan extensions, Lemma 1.2 provides adjunctions

$$\mathrm{Lan}_K(-) \dashv K^* \dashv \mathrm{Ran}_K(-).$$

**Exercise 3.4.** If $\mathcal{D}$ is cocomplete, then all $\mathrm{Lan}_K F$ exist and can be computed via

$$(\mathrm{Lan}_K F)(e) \cong \operatorname*{colim}_{Kc \longrightarrow e} Fc \quad (\forall e \in \mathcal{E})$$

where the colimit is over $(K \downarrow \mathrm{const}_e)$.

**Corollary 3.5.** *If $\mathcal{D}$ is cocomplete and $K$ is fully faithful, then $\mathrm{Lan}_K(-)$ is fully faithful.*

*Proof.* By Lemma 1.6 it suffices to show that each $\eta_F\colon F \Longrightarrow (\mathrm{Lan}_K F)\circ K$ is an iso. Since $K$ is fully faithful, we have $(K \downarrow \mathrm{const}_{Kx}) \cong \mathcal{C}_{/x}$. The latter has the terminal object $\mathrm{id}_x$, so taking the colimit corresponds to evaluating at this object:

$$(\mathrm{Lan}_K F)(Kx) \cong \operatorname*{colim}_{Kc \longrightarrow Kx} Fc \cong \operatorname*{colim}_{c \longrightarrow x} Fc \cong Fx$$

$\square$

**Lemma 3.6.** *Every presheaf is a colimit of representables. Let $F \in \mathrm{PSh}(\mathcal{C})$. Then*

$$F \cong \operatorname*{colim}_{\text{よ}(c) \longrightarrow F} \text{よ}(c)$$

*where we index over* $(\text{よ} \downarrow \mathrm{const}_F)$.

*Proof.* For every $G \in \mathrm{PSh}(\mathcal{C})$ we have

$$\mathrm{Hom}_{\mathrm{PSh}(\mathcal{C})}\Big(\operatorname*{colim}_{\text{よ}(c)\longrightarrow F} \text{よ}(c), G\Big) \cong \lim_{\text{よ}(c)\longrightarrow F} \mathrm{Hom}_{\mathrm{PSh}(\mathcal{C})}\big(\text{よ}(c), G\big) \overset{(\star)}{\cong} \mathrm{Hom}_{\mathrm{PSh}(\mathcal{C})}(F, G).$$

To see $(\star)$, note that $\mathrm{Hom}_{\mathrm{PSh}(\mathcal{C})}(F, G)$ has a canonical cone structure by precomposition:

$$\begin{tikzcd}
\text{よ}(x) \arrow[rr, "\text{よ}(f)"] \arrow[dr, "\alpha"'] & & \text{よ}(y) \arrow[dl, "\beta"] & & {[\text{よ}(x), G]} & & {[\text{よ}(y), G]} \arrow[ll, "\text{よ}(f)^*"'] \\
& F & \longmapsto & & & {[F, G]} \arrow[ul, "\alpha^*"] \arrow[ur, "\beta^*"'] &
\end{tikzcd}$$

$\square$

**Corollary 3.7.** *Let $F\colon \mathcal{C} \longrightarrow \mathcal{D}$. If $\mathcal{D}$ is cocomplete, then $\mathrm{Lan}_{よ} F$ has a right adjoint.*

*Proof.* By Lemma 1.2 it suffices to show that each

$$\mathrm{Hom}_{\mathcal{D}}(\mathrm{Lan}_{よ} F-, d)\colon\ \mathrm{PSh}(\mathcal{C})^{\mathrm{op}} \longrightarrow \mathbf{Set}$$

is representable. Using the Kan extension formula, we have

$$\begin{aligned}
\mathrm{Hom}_{\mathcal{D}}((\mathrm{Lan}_{よ} F)(X), d) &\cong \mathrm{Hom}_{\mathcal{D}}(\operatorname*{colim}_{よ(c)\longrightarrow X} Fc, d) \\
&\cong \lim_{よ(c)\longrightarrow X} \mathrm{Hom}_{\mathcal{D}}(Fc, d) \\
&\cong \mathrm{Hom}_{\mathrm{PSh}(\mathcal{C})}(X, \mathrm{Hom}_{\mathcal{D}}(F-, d))
\end{aligned}$$

so the right adjoint is given by the functor

$$R\colon\ \mathcal{D} \longrightarrow \mathrm{PSh}(\mathcal{C}), \quad d \longmapsto \mathrm{Hom}_{\mathcal{D}}(F-, d).$$

□

**Theorem 3.8.** *Suppose $\mathcal{D}$ is cocomplete. The Yoneda embedding $よ\colon \mathcal{C} \longrightarrow \mathrm{PSh}(\mathcal{C})$ induces a weak equivalence by restriction*

$$よ^*\colon\ \mathrm{Fun}^{\mathrm{colim}}(\mathrm{PSh}(\mathcal{C}), \mathcal{D}) \xrightarrow{\simeq} \mathrm{Fun}(\mathcal{C}, \mathcal{D})$$

*where $\mathrm{Fun}^{\mathrm{colim}}(\mathrm{PSh}(\mathcal{C}), \mathcal{D})$ is the full subcategory of $\mathrm{Fun}(\mathrm{PSh}(\mathcal{C}), \mathcal{D})$ spanned by the colimit preserving functors. Furthermore, every such colimit preserving functor admits a right adjoint.*

*Proof.* Since each $\mathrm{Lan}_{よ} F$ has a right adjoint, $\mathrm{Lan}_{よ} \dashv よ^*$ restricts to

$$\mathrm{Lan}_{よ}\colon\ \mathrm{Fun}(\mathcal{C}, \mathcal{D}) \leftrightarrows \mathrm{Fun}^{\mathrm{colim}}(\mathrm{PSh}(\mathcal{C}), \mathcal{D}) : よ^*.$$

Since $よ$ is fully faithful, so is $\mathrm{Lan}_{よ}$ by Corollary 3.5. Finally, $よ^*$ reflects isos on colimit preserving functors since every presheaf is a colimit of representables by Lemma 3.6. □

# 4 Simplicial sets

**Definition 4.1.** The simplex category $\Delta$ has the following data.

- Objects: totally ordered sets $[n] \coloneqq \{0 < 1 < \cdots < n\}$ (where $n \in \mathbb{N}_{\geq 0}$)

- Morphisms: monotone (i.e. weakly increasing) maps.

A simplicial set is a presheaf on $\Delta$, i.e. a functor $X\colon \Delta^{\mathrm{op}} \longrightarrow \mathbf{Set}$. The category of simplicial sets is defined as $\mathbf{sSet} \coloneqq \mathrm{PSh}(\Delta)$.

**Remark 4.2.** We can view $\Delta$ as a full subcategory of $\mathbf{Cat}$ by identifying $[n]$ with its posetal category.

**Remark 4.3.** The category $[n]$ gives a combinatorial description of an $n$-simplex with labeled vertices.

$$\begin{tikzcd} 0 \arrow[rr] & & 1 \end{tikzcd}$$

[1]

$$\begin{tikzcd} & 2 & \\ 0 \arrow[ur] \arrow[rr] & & 1 \arrow[ul] \end{tikzcd}$$

[2]

$$\begin{tikzcd} & 3 & \\ & 2 \arrow[u] & \\ 0 \arrow[uur] \arrow[ur] \arrow[rr] & & 1 \arrow[ul] \arrow[uul] \end{tikzcd}$$

[3]

- For each $i \in [n]$ there is a unique injective morphism

$$d^i\colon [n-1] \rightarrowtail [n]$$

  that does not hit $i$, called the $i$-th coface map.

- For each $j \in [n-1]$ there is a unique surjective morphism

$$s^j\colon [n] \twoheadrightarrow [n-1]$$

  that hits $j$ twice, called the $j$-th codegeneracy map.

For $X \in \mathbf{sSet}$ we introduce the following notation.

- $X_n \coloneqq X([n]) \in \mathbf{Set}$ are called the $n$-simplices of $X$.
- $d_i \coloneqq X(d^i)\colon X_n \longrightarrow X_{n-1}$ are called the face maps of $X$.
- $s_j \coloneqq X(s^j)\colon X_{n-1} \longrightarrow X_n$ are called the degeneracy maps of $X$.

$x \in X_n$ is called degenerate, if $x \in \mathrm{im}(s_j)$ for some $j$.

**Definition 4.4.** The (standard) $n$-simplex is defined as $\Delta^n \coloneqq \text{よ}([n]) = \mathrm{Hom}_\Delta(-,[n]) \in \mathbf{sSet}$. We introduce the following sub-simplicial sets.

- $\partial\Delta^n := \bigcup_{i=0}^{n} \operatorname{im}(よ(d^i)\colon \Delta^{n-1} \longrightarrow \Delta^n)$, the boundary of $\Delta^n$,

- $\Lambda^n_j := \bigcup_{\substack{i=0 \\ i\neq j}}^{n} \operatorname{im}(よ(d^i)\colon \Delta^{n-1} \longrightarrow \Delta^n)$, the $j$-horn in $\Delta^n$

Furthermore, we call $\Lambda^n_j$ an

- inner horn, if $0 < j < n$,
- outer horn, if $j = 0$ or $j = n$.

**Definition 4.5.** Let $|\Delta^n| \in \mathbf{Top}$ be the geometric standard simplex, given by the convex hull of the standard basis vectors $e_0, e_1, \dots, e_n \in \mathbb{R}^{n+1}$. Given $f\colon [m] \longrightarrow [n]$, we obtain

$$|f|\colon |\Delta^m| \longrightarrow |\Delta^n|$$

by linearly extending $e_i \longmapsto e_{f(i)}$. This defines a functor $\Delta \longrightarrow \mathbf{Top}$. By Theorem 3.8 this has a unique colimit-preserving extension, the *geometric realization*

$$|\cdot|\colon \mathbf{sSet} \longrightarrow \mathbf{Top}, \quad |X| \cong \operatorname*{colim}_{\Delta^n \longrightarrow X} |\Delta^n|$$

$$\begin{tikzcd} \Delta \arrow[r] \arrow[d, "よ"'] & \mathbf{Top} \arrow[dl, shift left, "\mathrm{Sing}"] \\ \mathbf{sSet} \arrow[ur, shift left, "|\cdot|"] & \end{tikzcd}$$

whose right adjoint must be the *singular complex* functor

$$\mathrm{Sing}\colon \mathbf{Top} \longrightarrow \mathbf{sSet}, \quad \mathrm{Sing}(X) = \mathrm{Hom}_{\mathbf{Top}}(|\Delta^{(-)}|, X).$$

**Exercise 4.6.** Show that the geometric realization of a simplicial set has a canonical CW-complex structure.

**Definition 4.7.** Every (small) category $C$ defines a simplicial set

$$\mathrm{N}(C)\colon \Delta^{\mathrm{op}} \hookrightarrow \mathbf{Cat}^{\mathrm{op}} \xrightarrow{\mathrm{Hom}_{\mathbf{Cat}}(-,C)} \mathbf{Set}$$

called the *nerve* of $C$. By Theorem 3.8 we get an adjunction

$$h\colon \mathbf{sSet} \leftrightarrows \mathbf{Cat} : \mathrm{N}$$

where $h$ is the unique colimit preserving extension of $\Delta \hookrightarrow \mathbf{Cat}$. We call $h$ the homotopy category functor.

**Definition 4.8.** For each $X \in \mathbf{sSet}$, the functor $(-)\times X\colon \mathbf{sSet} \longrightarrow \mathbf{sSet}$ preserves colimits (because it does so pointwise). By Theorem 3.8 it must be the extension of

$$\Delta \longrightarrow \mathbf{sSet}, \quad [n] \longmapsto \Delta^n \times X.$$

We obtain an adjunction

$$(-)\times X\colon \mathbf{sSet} \leftrightarrows \mathbf{sSet} : \mathrm{F}(X,-)$$

where $\mathrm{F}(X,Y)_n \cong \mathrm{Hom}_{\mathbf{sSet}}(\Delta^n \times X, Y)$ defines the internal hom in **sSet**, also called the function complex.

# References

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[Wag25] Ferdinand Wagner. "∞-Categories in Topology". Available at https://florianadler.github.io/inftyCats/inftyCats.pdf. Sept. 2025.